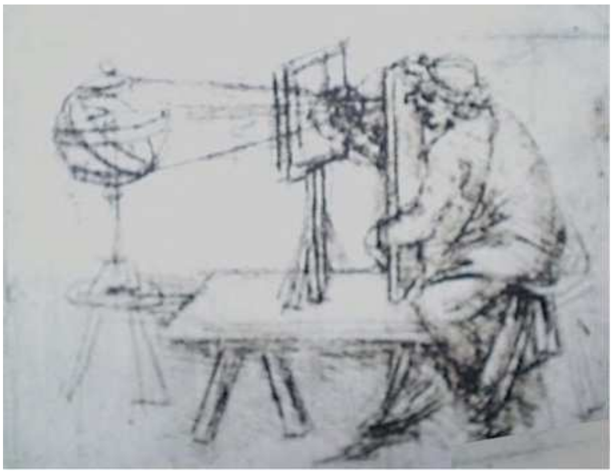

Figure 1. *Drawing of a man using a perspectograph to draw an armillary sphere. Leonardo da Vinci (Codex Atlanticus I, 5r; ~1480)*

The second link to a larger scale conception is the presence of Paolo di Pozzo Toscanelli in the Florence of da Vinci's youth. Toscanelli had been one of the progenitors of perspective geometry in discussions with Brunelleschi early in the century [[iii]], then worked with Nicolas di Cusa to produce the classic Renaissance text on squaring the circle [[iv]] and other philosophical matters before turning his attention to cartography. As a cartographer, he was in demand well beyond the shores of Italy, providing maps of the Mediterranean to rulers such as King John of Portugal in 1460. Moreover, as an astronomer, Toscanelli detailed his observations on what later became known as Halley's comet, three centuries before Edmund Halley's notation of it. Importantly, as an astronomer, he took the lead in installing the world's first astronomical gnomon in the Florence Duomo, a hole near the top of the dome that was incorporated into a *camera-obscura*-type installation to track the movements of the sun and determine the timing of movable festivals such as Easter. The unveiling of this remarkable intersection of science and religion took place to great public fanfare in 1468. As chief pupil of the workshop of Andrea Verrocchio, the heir to Donatello and Michelozzo as the principal artisanal provider to the ruling Medici clan, da Vinci would certainly have been present as one of the supporting cast at this momentous event, and may even have performed musically to celebrate it (as he was famous in his youth for composing extempore songs at public events).Da Vinci himself has a close connection with the Duomo at that time, when he was an impressionable young man in his late teens. In the same year, 1468, his master

completed by Michelozzo in 1461, and the 8 tonne gilded copper ball was added by Verrocchio and Leonardo da Vinci in 1469.

Figure 3. *Left: 'Crying Heraclitus and Laughing Democritus' by Bramante (1483). Right: 'The Philosopher (a possible portrait of Leonardo da Vinci with a globe)' by Michelangelo (~1515).*

A further link of da Vinci to global cartography is found in a fresco by his friend Donato Bramante from his early years in Milan (Figure 3, left). The painting depicts the Greek philosophers Heraclitus and Democritus, representing pessimism and optimism, respectively, flanking a geographically detailed depiction of the globe. The two figures are generally accepted as depictions of da Vinci on the left and Bramante himself, on the right. Their association with the globe seems to reflect a mutual interest in the large-scale conception of the land they inhabited, consistent with the idea that da Vinci brought the global interests that he had developed in Florence with him to Milan when he relocated there in 1482. The final association comes in a sketch by Michelangelo of a philosopher holding a globe (Figure 3, right) that is often considered to be a portrait of the aging Leonardo da Vinci (especially as it seems to be the model for Vasari's portrait of him in his *Vitae*). The fact that the philosopher is holding a globe has nevertheless been considered puzzling in relation to this attribution.

DA VINCI'S WORLD MAP

Despite Leonardo da Vinci's renown as a Renaissance man, and as a cartographer in particular, he is not generally acknowledged as authoring a world map (or *mappamundi*)

Arctic Ocean and the landmass of Antarctica (at the centers of the right and left florets, respectively), something achieved by no other cartographer of the era, or for the next two centuries! The Arctic is particularly surprising in view of the fact that it is continuously covered in a thick layer of ice.

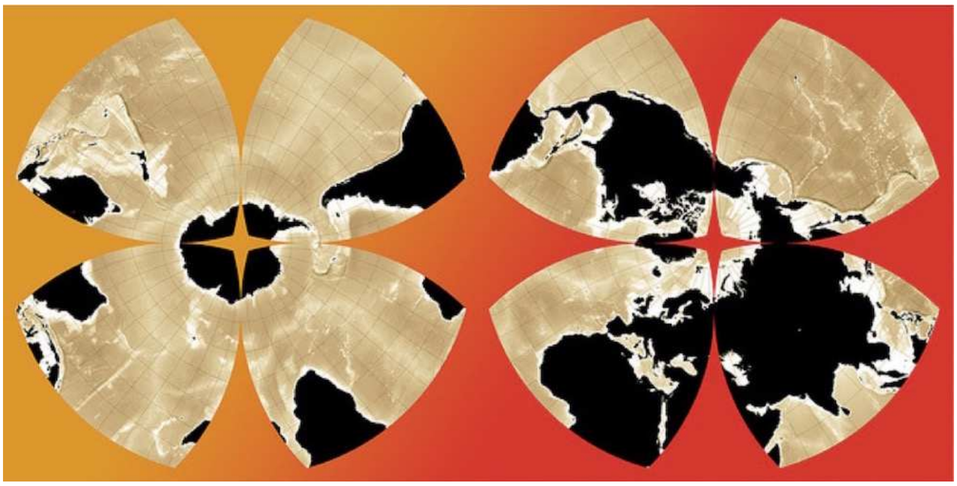

Figure 5. *Modern projection of the globe onto da Vinci's octant projection, based on a bathymetric image of the ocean depths from NASA Blue Marble [vi].*

In relation to Antarctica, it may be relevant that the Portuguese explorer Tristão di Cunha, who first discovered the eponymous island of Tristan di Cunha in 1504, was the Portuguese ambassador to Rome in 1514. Given that this was during the period of da Vinci's residence in Rome (1513-1516), and his intensive interest in mapping leading up to this period, it seems almost certain that he would have met with di Cunha to have discussions about the geography of the Southern Hemisphere and may have gleaned some little known hints of an Antarctic continent from him (such as patterns of bird migration, for example).

ATTRIBUTION TO LEONARDO DA VINCI

Despite the fact that it was found in his collection of papers from those assembled by his pupil Francesco Melzi after his death, the attribution of this *mappamundi* to Leonardo da Vinci has long been questioned by most da Vinci scholars [vii, viii, ix], and has consequently been omitted from almost all books in his oeuvre (the sole exception being a passing

Figure 6. *Map of the eastern Mediterranean and drawing of the tower of the Palazzo della Signoria (Codex Atlanticus, 1106r, 1496).*

With respect to his actual authorship, it is interesting to note that da Vinci's cartographic style is substantially different from his loose and flowing drawing style. His cartographic style can be seen in his map of the eastern Mediterranean (Figure 6), which is also one of the few of his maps known beyond the borders of Italy, helping to underline the global reach of his cartographic interests. This accepted autograph map helps to link the *mappamundi* to da Vinci in two ways. The line style can be seen to have the same curly character and the lettering to have a similar plain character to those in the *mappamundi*, although the letters are predominantly lower case as in most of his maps. The switch to upper case can be hypothesized to be indicative of an intention to provide the *mappamundi* to a sponsor of some kind, such a Cesare Borgia or Giuliano de Medici, both of who

sheet to the conception underlying the *mappamundi*. Taken together with Figure 6, this notebook page seems to remove any grounds for questioning da Vinci's authorship of the unique *mappamundi*. Even if it is in fact a copy by a pupil, the pupil is most unlikely to have the kind of knowledge required to make major modifications, so we are safe in assuming that all the characteristic details are reflective of da Vinci's own intentions.

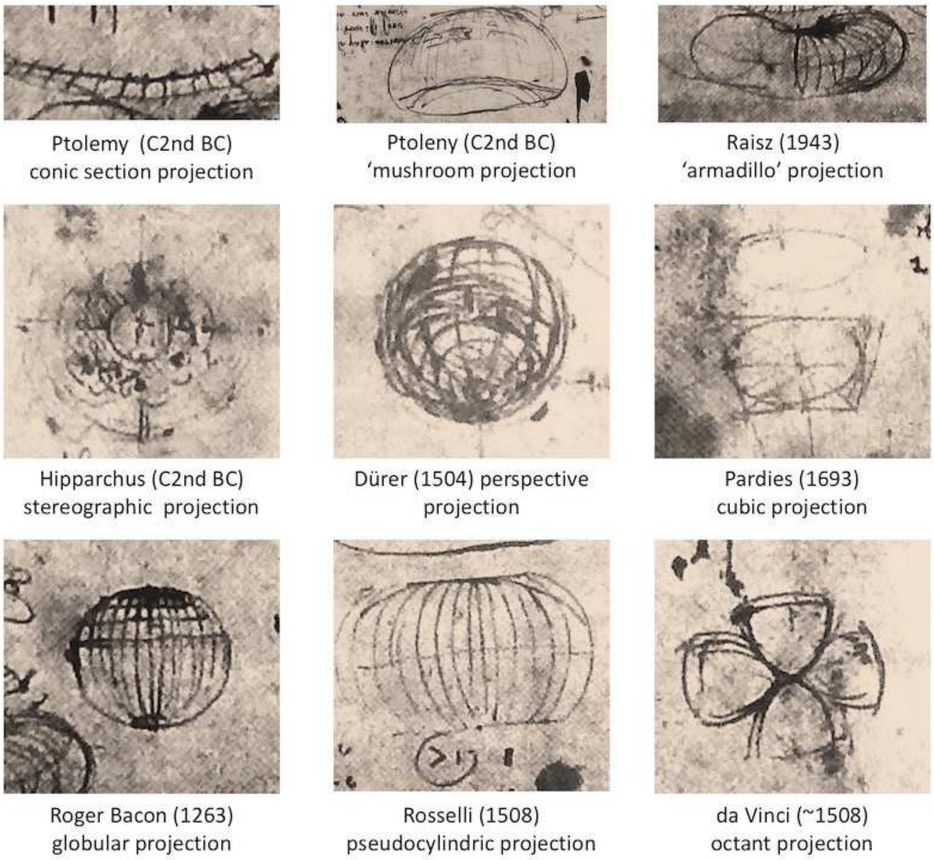

Figure 8. *Identification of the precursors or earliest known examples of nine different projections explored by Leonardo da Vinci in his notebook page. For those projections dated later than 1508, his drawings should be effectively considered the original precursors.*

To highlight the variety of projections considered in this notebook page, they have been isolated in Figure 8 and annotated with the originator of each form of projection (as far as they are known). Three of them date back to Roman times, as represented by

Da Vinci was evidently aware of the geometric significance of this figure, as he develops it as means of constructing a regular pentagon (Figure 9, right). If connected at one apex, equilateral triangles would form a hexagonal figure. To employ them to construct a pentagonal shape, da Vinci has subdivided the height into five subdivisions and indicated that the topmost division should be the center of pentagon. This is a very good approximation to a valid construction, because the angle at the center derived in this manner is 71.6° instead of the required 72°, which is accurate to 99.44%. Whether this pentagonal construction relates to other geometric figures that he developed, such as the Vitruvian Man, is unclear.

GEOMETRIC DEVELOPMENT

Da Vinci is renowned for the range of geometric explorations in his notebooks. One such development that relates to his octant approach to world mapping is that of 'squaring the circle', which has already been noted as a key interest of Toscanelli [4]. Da Vinci's interest is often referred to his iconic illustration of the geometric relationships in human proportions developed by the Roman architect, Vitruvius, known as the Vitruvian Man. Since, however, those proportions and the configuration of the elements were indeed worked out by Vitruvius, it is not clear from this what level of understanding da Vinci needed to have had in order to construct this diagram. In this connection, therefore, it is of interest to find explicit analyses of the relationships between the circle and its circumscribing square in his notebook pages, such as that reproduced in Figure 10. Note that this figure also provides a link through the obliques drawn through the circle, which prefigure the arm and leg positions to the Vitruvian Man geometry.

At lower left, he develops an octagonal analysis of the area of a circle inscribed in a square rhombus (regular diamond shape) by approximating it with the eight triangles making up an inscribed octagon, reconfigured as an 'accordion' figure of eight isosceles triangles (or, actually, seven isosceles triangles and two right triangles making up the eighth, labeled *abcdefghi*) together with a lenticular segment labeled *K* to account for the eight missing lenticular elements in the octagonal approximation to the circle. This large lenticular segment *K* should have an area eight times larger than that of each of the eight segments in the circular figure, giving it a linear extent of sqrt(8), or 2*sqrt(2). Deriving this ratio seems to be the motivation for the scaled figures of the semicircle, quadrant, octant and sector, which all fill the same area relative to the original circle, making it clear that the lenticular segment is in the correct ratio of sqrt(8)/4 = 0.707 to the width of the eight triangle figure *abcdefghi*.

later. Moreover, some of the approaches on these pages involve explicit visualization of the spherical octant as a Reuleaux-like spherical triangle, which may well have been what suggested the Reuleaux triangle approach to Da Vinci for his world map.

Figure 11. *Octahedral analysis of the volume of a sphere with various forms of segmentation (Leonardo da Vinci, Paris Manuscript E 24v,25r).*

ROLE IN THE DISCOVERY OF AMERICA

This paper began with the evidence that da Vinci had a global conceptualization of the world in his youth, and would most likely have encountered Paolo Toscanelli in his role in the leading *bottega* in Florence working at the Duomo at the same time as Toscanelli's gnomon. Now, as a cartographer, Toscanelli was in communication with the authorities in Portugal and Spain, and was reputed to have developed the (erroneous) map that inspired Columbus to set out to discover the western route to the Indies (which were known from the overland route through Asia). Toscanelli lived to 1482 (about the time that da Vinci left Florence), so his map must have been drawn at least a decade before Columbus set sail, and in the period when he would have known da Vinci. This gives rise to the speculation that da Vinci may have been involved in the conversation about the extension of the Ptolemaic octant (for that is about how much of the globe was mapped in Ptolemy's recently popularized Geographia) to the remainder of the full globe.

LATER INFLUENCES

Although barely recognized in the canon of cartographic projections, da Vinci's octant projection can be seen to have influenced a line of cartographers to the present day. The first example of this influence was the diagrams of Oronce Fine (1494-1555), a French cartographer active in the years just after da Vinci died in France, who was notable for 'crossing swords' on academic matters with King François I, and who developed the gridlines of the spherical octant projection and elaborated it into several formats, including a whimsical heart-shaped projection (Figure 13). There was a further flurry of interest in octant maps the following century when Daniel Angelocrator (Engelhardt) produced several rigorous versions [[xiv]]. (Unlike da Vinci, all these successors had Antarctica dramatically too large, however.)

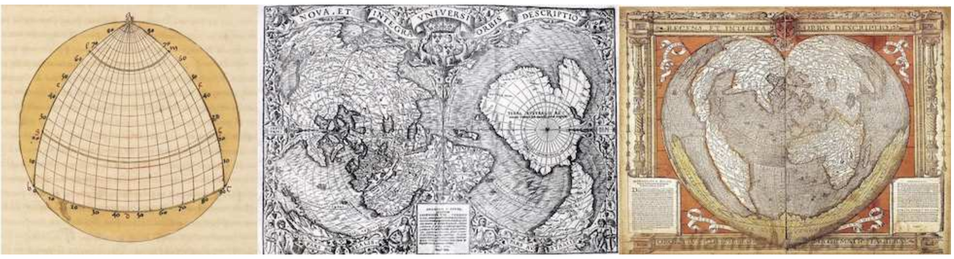

Figure 13. *Developments of the spherical octant projection by Oronce Fine (1542, 1531, 1536).*

We may jump forward several centuries to the Pan-Pacific Exposition of 1915 announcing the recovery of San Francisco from the great earthquake of 1906, for which Bernard J. Cahill [[xv]] developed a connected version of the da Vinci octant projection to map the challenge of a flight around the world (Figure 14). How da Vinci would have appreciated this combination of two of his lifelong interests! To provide more extended connection regions, Cahill slightly distorted the da Vinci octants into figures with each edge consisting of three straight segments, then connected the four pairs of north-south quasi-octants into a butterfly-shaped figure that allowed all the continents except Asia and Antarctica to appear as connected landmasses. Many of the continents are rotated to extreme angles relative to their natural north-south axes, however.

Following Cahill, several cartographers developed further forms of segmented maps, by projection onto various regular polyhedral, notably the Buckminster Fuller octahedron (and the other regular polyhedra) and the Keyes projection that took a different configuration of the Cahill projection to avoid the cut through central Asia (but still lopped off the Russian Far East peninsula and segregated New Zealand from

Australia, however). The Keyes projection also had continents such as Africa tilted at a rather severe angle.

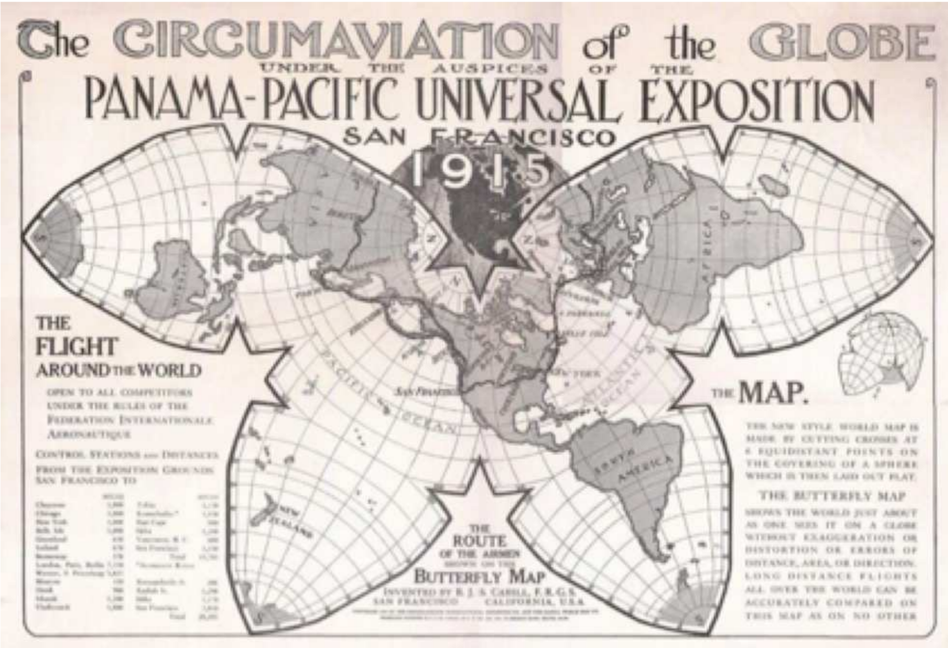

Figure 14. *Connected quasi-octant map by Bernard J. Cahill (1909).*

These deficiencies may be partially rectified with a Cahill-like projection that does not adhere to the principle of complete octants, but allows segments of the octants to be shifted to complete the continents and also to regularize the oceans somewhat. A final map taking this approach, developed by the present author, is depicted in Figure 15. Part of the motivation for including this reconfigured projection is to illustrate how powerful the da Vinci approach was in providing for closer-to-isotropic projection of the relative shapes and sizes of all countries and continents throughout the globe. All other projections distort the shapes of at least one continent, and most projections vastly exaggerate the sizes of Greenland and Antarctica. Here one can clearly see the approximate equivalence in size of the United States, Brazil, China, Australia and Antarctica, for example. Distances, too, are well represented by straight lines on the map within continents. The main sacrifice is of distances across oceans, which are heavily distorted in this projection. It may be possible, however, to derive an equivalent projection preserving the shapes and size of the oceans in the same way, although it is likely to be more challenging because they cover the majority of the earth's surface.

Figure 15. *Da Vinci-Cahill-Tyler quasi-isotropic world map, reconfigured from the Cahill projection to retain all continents as contiguous landmasses minimally distorted in shape and relative sizes.*

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REFERENCES

[i] Rosselli F (1508) In: Almagià R. (1951) On the cartographic work of Francesco Rosselli. *Imago Mvndi.* International Society for the History of Cartography 8: 27-34.
[ii] Veltman KH (1986) *Studies on Leonardo da Vinci I: Linear Perspective and the Visual Dimensions of Science and Art.* Deutscher Kunstverlag: Berlin.
[iii] Vasari G (1550) *Le Vite de' piú Eccellenti Architetti, Pittori, et Scultori Italiani, da Cimabue Insino a' Tempi Nostri. Nell'edizione per i tipi di Lorenzo.* Torrentino:Firenze.
[iv] di Cusa N (with Toscanelli P di P) *Dialogus de circuli quadratura.* Manuscript, 1457.
[v] "In the Codice Atlantico we find sketches of a globe being unfolded that is not far from Waldseemüller's pseudo-Ptolemaic, cordiform projection (1507)." Veltman (1986).
[vi] Burchill L (2007) *Leonardo's octant map.* https://www.flickr.com/photos/lloydb/529393602/
[vii] Major RH (1865) *Memoir on a Mappemonde by Leonardo da Vinci, Being the Earliest Map Hitherto Known Containing the Name of America: Now in the Royal Collection at Windsor.* J.B. Nichols: London.
[viii] Nordenskjöld AE (1889) *Facsimile-Atlas to the Early History of Cartography with Reproductions of the Most Important Maps Printed in the XV and XVI Centuries.* Trans. Johan Adolf Ekelöf (Stockholm, 1889; reprinted, New York: Dover, 1973).
[ix] Snyder JP (1993) *Flattening the Earth: Two Thousand Years of Map Projection.* University of Chicago Press: Chicago.
[x] Cecchi A (2003) New light on Leonardo's Florentine patrons. In, Bambaugh C, Stern R, Manges A (eds) *Leonardo Da Vinci Master Draftsman.* Metropolitan Museum of Art: New York, 121-139.
[xi] Möller E (1937-8) Leonardos Bildnis der Ginevra dei Benci. *Münchner Jahrbuch der bildenden Kunst.* n.s. 12, 185-209.
[xii] Bambach C, Stern R, Manges A.(2003) Leonardo Da Vinci Master Draftsman: Catalogue to an Exhibition at The Metropolitan Museum of Art. *Metropolitan Museum of Art* (New York, N.Y.), p. 131.
[xiii] Reuleaux F (1876) *The Kinematics of Machinery: Outlines of a Theory of Machines.* London: Macmillan.
[xiv] Angelocrator D (1628) *Doctrina de ponderibus, monetis, et mensuris per totum terrarum orbem usitatis.* Johann Berner: Frankfurt.
[xv] Cahill BJS (1909) An account of a new land map of the world. *The Scottish Geographical Magazine,* 1909-09, 449-469.